MA3 – WEEK 2

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1. Examples

Example 1.1. Show that

(1)
$$\mathbb{P}[A \cap (B \cap C)^c] = \mathbb{P}[A] - \mathbb{P}[A \cap B \cap C]$$

(2)
$$\mathbb{P}[A \cap B \cap C] = \mathbb{P}[A|B \cap C] \cdot \mathbb{P}[B|C] \cdot \mathbb{P}[C]$$

(3) if $\mathbb{P}[A] > 0$, then
$$\mathbb{P}[A \cap B|A] \geq \mathbb{P}[A \cap B|A \cup B].$$

(4)
$$\mathbb{P}[A|B] = \frac{\mathbb{P}[B|A]\mathbb{P}[A]}{\mathbb{P}[B|A]\mathbb{P}[A] + \mathbb{P}[B|A^c]\mathbb{P}[A^c]}.$$

(5) if A and B are independent, then A and B^c are also independent.

Example 1.2. A man has n keys on a key ring, one of which opens the door to his apartment. He asked one of his friends to pick up something from his apartment without telling his friend which key was the apartment key. His friend came up with a plan. He will choose a key at random and try it. If it fails to open the door, he will discard it and choose one of the remaining $n - 1$ keys randomly. Show that the probability that the door opens with the third key he tries equals that of the first key.

Example 1.3. Alice and Bob are playing a game where they both roll a die and the higher roll wins.

(1) Alice rolls a 4. What is the probability she wins?
(2) What is the probability that someone wins?
(3) Is the probability that someone wins independent of
  (a) what Alice/Bob rolls?
  (b) of both their rolls?
  (c) of the sum of their rolls?
(4) Suppose Alice won. What is the probability she rolled a 4?

Example 1.4. An urn contains w white chips, b black chips, and r red chips. The chips are drawn out at random, one at a time, with replacement. What is the probability that a white appears before a red?