# MA3 – WEEK 1

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## 1. Examples

**Example 1.1** (Permutation)**.** How many different 7-place Californian license plates are possible? (1-number, 3 consecutive letters and 3 consecutive numbers)

**Example 1.2** (Dividing into distinctive groups)**.**

(1) How many possible ways to arrange the word "Wollongong"?
(2) The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge deals are possible?
(3) What is the coefficient of $w^2x^3yz^3$ in the expansion of $(w+x+y+z)^9$?

**Example 1.3** ("Balls in urns")**.** Nine students, five men and four women, interview for four summer internships sponsored by a city newspaper.

(1) In how many ways can the newspaper choose a set of four interns?
(2) In how many ways can the newspaper choose a set of four interns if it must include two men and two women in each set?
(3) What is the probability that out of 4 interns that have been chosen randomly, not everyone is of the same sex?

**Example 1.4** (Placement problem)**.** There are 5 seats in a row in a movie theater. You know that two of your friends just had a fight and they do not want to sit together for the duration of the movie. How many possible ways can you assign seats for your 5 friends?

**Example 1.5** (Cards problems)**.** In a standard deck of 52 cards (perfectly shuffled)

(1) what is the probability that the $k$-th card is an queen?
(2) what is the probability that the first queen appears in the $k$-th place, where $k \leq 52$?
(3) a bridge hand (thirteen cards) is dealt. Let $A$ be the event that the hand contains four aces; let $B$ be the event that the hand contains four kings. Find $P(A \cup B)$.
(4) two cards are distributed to each of three players. What is the probability that at most one player has one ace and one king? (hint: exclusion-inclusion principle)

**Example 1.6** (Dice problem)**.**

(1) Five fair dice are rolled. What is the probability that the faces showing constitute a "full house"—that is, three faces show one number and two faces show a second number?
(2) Roll a fair dice until a 5 or 6 comes up. What is the probability that we see 5 before 6?